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SCATTERING OF ELECTRONS BY A SCREENED COULOMB FIELD IN HIGHER BORN APPROXIMATION

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ABSTRACT. The relativistic scattering of electrons by heavy atoms has been studied by taking the Rozental approximation of the Thomas–Fermi potential. The differential cross section of scattering has been calculated in the Born approximation up to second order in the expansion of $\frac{Ze^2}{\hbar v}$. The numerical results of the scattering of electrons of energy 150 KeV at an angle 90° have been given for the elements, argon, krypton, xenon and mercury.

INTRODUCTION

We propose to study the scattering of the fast electrons by heavy atoms. For high energy scattering the potential of the atom is usually taken as that due to the nuclear charge alone, the screening effects of the atomic electrons being neglected. It is, however, worth while to investigate the effects of screening due to the atomic electrons on electron scattering. The Hartree-Fock method which takes good account of the screening in the potential function is suitable only for light atoms, considering the practical difficulties involved. For heavy atoms the treatment of the screened potential by the statistical method of Thomas-Fermi is more practicable. Unfortunately there is no analytic expression for the Thomas-Fermi potential. Various approximate representations of the Thomas-Fermi potential are cited in the literature (Majewisky and Tietz, 1957). Among them the Rozental form which is a combination of three potentials of Yukawa type with different weight factors, is suitable for analytic treatment in higher Born approximations. With this potential the differential cross section for electron scattering up to second order has been calculated. Numerical results are given for the elements argon ($Z = 18$), krypton ($Z = 36$), xenon ($Z = 54$) and mercury ($Z = 80$) at 150 KeV incident electron energy. These are compared with the corresponding numerical results for the Coulomb potential without screening. The differential scattering cross section are expressed as the ratio of $d\sigma$ to $d\sigma_R$ where $d\sigma_R$ is the well-known Rutherford scattering cross section.

RESULTS AND DISCUSSION

Taking relativistic units $\hbar = c = m = 1$, the Dirac equation can be written as

$$[E - i(\alpha \cdot \nabla) + \beta]\psi = -Ze^2 V\psi$$

$$V = \frac{1}{r}\phi(r) = \frac{1}{\mu x}\,[0.255e^{-.246x} + 0.581e^{-.947x} + 0.164e^{-4.356x}]$$

where

$$x = \frac{r}{\mu}, \quad \mu = \frac{1}{4}\left(\frac{9\pi^2}{2Z}\right)^{1/3} a_0, \quad a_0 = \text{First Bohr radius}$$

Following Vachaspati (1954), the expression for the scattering cross section in Born approximation up to second order can be written as

$$d\sigma = d\sigma_1 + d\sigma_2$$

where $$d\sigma_1 = 4(Z\alpha)^2 |a_1|^2 k^2 \cos^2\theta/2 \left(1 + \frac{1}{k^2}\sec^2\theta/2\right)$$

$$d\sigma_2 = 16(Z\alpha)^3 a_1 E k^2 \cos^2\theta/2 \left[(a_{3r} + \tfrac{1}{2}a_{2r}) + \frac{1}{k^2} a_{2r} \sec^2\theta/2\right]$$

$$a_1 = \frac{1}{4\pi}(\mathbf{k}_1 | V | \mathbf{k}_0), \qquad \mathbf{k}_0 = \text{initial momentum}, \qquad \mathbf{k}_1 = \text{final momentum}$$

$$|\mathbf{k}_0| = |\mathbf{k}_1| = k$$

$$a_{2r} = \frac{1}{4\pi(2\pi)^3} P\int \frac{(\mathbf{k}_1 | V | \mathbf{k}')(\mathbf{k}' | V | \mathbf{k}_0)}{k'^2 - k^2} d^3k'$$

$$= \frac{4\pi}{(2\pi)^3} \sum_{i<j=1}^{3} \frac{\alpha_i\alpha_j}{k^3} M_3(\lambda_i,\lambda_j) + \frac{4\pi}{(2\pi)^3} \sum_{i=1}^{3} \frac{\alpha_i^2}{k^3} M_3(\lambda_i, \lambda_i)$$

$$a_{3r} = \frac{1}{4\pi(2\pi)^3(n, P)} P\int \frac{(\mathbf{k}_1 | V | \mathbf{k}')(n \cdot \mathbf{k}')(\mathbf{k}' | V | \mathbf{k}_0)}{k'^2 - k^2} d^3k'$$

$$= \frac{4\pi}{(2\pi)^3 P^2} \sum_{i<j=1}^{3} \left\{ \frac{4k^2 + \lambda^2_i + \lambda^2_j}{k^3} M_3(\lambda_i, \lambda_j) + \frac{2}{k} M_2(\lambda_i, \lambda_j) - I_i - I_j \right\} \alpha_i\alpha_j$$

$$+ \frac{4\pi}{(2\pi)^3 P^2} \sum_{i=1}^{3} \left\{ \frac{2k^2 + \lambda_i^2}{k^3} M_3(\lambda_i, \lambda_i) + \frac{1}{k} M_2(\lambda_i,\lambda_i) - I_i \right\} \alpha_i^2$$

M_2 and M_3 are given by Lewis (1956) and quoted in the Appendix.

$\alpha_1, \alpha_2, \alpha_3 = 0.255, 0.581, 0.164$; $\lambda_1, \lambda_2, \lambda_3 = 0.246, 0.947, 4.356$.

The Rutherford scattering cross section is $d\sigma_R = \frac{(Ze^2)^2}{4} \frac{1-\beta^2}{\beta^4 \sin^4 \theta/2}$. The value of k for 150 KeV incident electron energy is 0.820534 and the ratios $d\sigma_1/d\sigma_R$ and $d\sigma_2/d\sigma_R$ are given by

$$\frac{d\sigma_1}{d\sigma_R} = \frac{2k^4 (k^2+2)}{(k^2+1)} |a_1|^2$$

$$\frac{d\sigma_2}{d\sigma_R} = 8Ze^2a_1E \frac{k^6}{k^2+1} \left[a_{3r} + \left(0.5 + \frac{2}{k^2}\right)a_{2r}\right]$$

TABLE I

Z	$d\sigma_1/d\sigma_R$		$d\sigma_2/d\sigma_R$		$d\sigma/d\sigma_R$	
	Coulomb	Rozental	Coulomb	Rozental	Coulomb	Rozental
18	0.798815	0.796772	0.054211	0.061621	0.853026	0.858393
36	0.798815	0.795642	0.108422	0.127284	0.907237	0.922926
54	0.798815	0.794643	0.162634	0.194867	0.961449	0.989510
80	0.798815	0.793438	0.240939	0.294507	1.039754	1.087945

In Table I, the valus of $\frac{d\sigma_1}{d\sigma_R}$ i.e., the ratio of the first order relativistic scattering cross section calculated with the Rozental potential and the Coulomb potential, to the Rutherford cross section (non-relativistic Coulomb scattering cross section) are given at 150 KeV incident electron energy for different values of the atomic number Z. Correspondingly, the ratio $\frac{d\sigma_2}{d\sigma_R}$, shows the contribution of the second order term only in the scattering cross section and finally there is a column for $d\sigma/d\sigma_R$ which is equal to $\frac{d\sigma_1 + d\sigma_2}{d\sigma_R}$. It is found that $d\sigma_1/d\sigma_R$ for the Rozental potential is slightly less (e.g. about 0.5% less for $Z = 54$) than the corresponding ratio for Coulomb potential; the difference between the two ratios increases as Z increases. This is quite in agreement with the expectation that the screening should decrease the scattering cross section, though the very small difference between the two ratios indicates that the effect of screening is not appreciable at this energy. It is interesting to note, however, that the ratio for the Rozental potential is considerably larger than the corresponding ratio for the Coulomb potential (e.g. about 20% larger for $Z = 54$). We can obtain $d\sigma_2$ for the Coulomb potential from the Rozental potential, after putting $\lambda_i = 0$, $i = 1, 2, 3$ and $\alpha_i = \alpha_j = 0$, $\alpha_k = 1$, then a_{2r} reduce to zero in this limit, and a_{3r} gives a non-zero value which correctly reduces $d\sigma_2$ for the Rozental case to the corresponding expression for the Coulomb case. In the Rozental case, however, the contribution to the cross section from the terms associated with a_{2r} is not negligible; in fact it is found to be about twenty per cent of the contributions due to the other terms.

APPENDIX

In our calculations we have utilised the following expressions given by Lewis (1956):

$$Re\, M_2(\mu, \nu) = Re\{k\int d^3k'[(K_1^2+\mu^2)(K_2^2+\nu^2)]^{-1}\} = \frac{2\pi^2 k}{K} \text{ arc tan } \left[\frac{K}{\mu+\nu}\right]$$

$$Re\; M_3(\mu, \nu) = Re\{k^3 \int d^3k'[(k'^2-k^2-i\epsilon)(K_1^2+\mu^2)(K_2^2+\nu^2)]^{-1}\}$$

$$=\pi^2k^3[k^2(K^2+\mu^2+\nu^2)^2-\rho^2\mu^2\nu^2]^{-\frac{1}{2}}|$$

$$\left\{ \text{arc tan} \left[\frac{k[K^2+(\mu+\nu)^2]+[k^2(K^2+\mu^2+\nu^2)^2-P^2\mu^2\nu^2]^{-\frac{1}{2}}}{\mu\nu(\mu+\nu)} \right] \right.$$

$$\left. -\text{arc tan} \left[\frac{k[K^2+(\mu+\nu)^2]-[k^2(K^2+\mu^2+\nu^2)^2-P^2\mu^2\nu^2]^{\frac{1}{2}}}{\mu\nu(\mu+\nu)} \right] \right\}$$

$$Re\; M_3(\mu, \mu) - 2\pi^2k^3\{[k^2K^2+4k^2\mu^2+\mu^4]^{-\frac{1}{2}}/K\}\times$$

$$\text{arc tan } \{K\mu[k^2K^2+4k^2\mu^2+\mu^4]^{-\frac{1}{2}}\}$$

$$Re \cdot I_i = Re\{ \int d^3k'[(K_1^2+\lambda_i^2)(k'^2 - k^2-i\epsilon)]^{-1}\} - \frac{\pi^2}{k} \text{ arc tan } \frac{2k}{\lambda_i} \text{ with.}$$

$$\boldsymbol{K} = \mathbf{k}_i-\mathbf{k}_f = \mathbf{k}_o-\mathbf{k}_1\;;\; \boldsymbol{P} - \mathbf{k}_o+\mathbf{k}_1\;;\; \boldsymbol{K}_1 = \mathbf{k}_o-\mathbf{k}'\;;\; \boldsymbol{K}_2 = \mathbf{k}'-\mathbf{k}_1.$$

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